We have

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2},$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

While you know how to derive these formulas or memorize the results, the following scenario may occur.

(1)

$$\sum_{k=2}^{n} k = ?$$

Solution. Not much to worry here. What is missing from this sum compared to $\sum_{k=1}^{n} k$, which has a known formula?

$$\sum_{k=2}^{n} k = (2+3+4+\cdots+n)$$

whereas

$$\sum_{k=1}^{n} k = (1+2+3+\cdots+n)$$

so we are missing the number 1. Therefore, you can simply write

$$\begin{aligned}\sum_{k=2}^{n} k &= \left(\sum_{k=1}^{n} k\right) - 1 \\ &= \frac{n(n+1)}{2} - 1 \\ &= \frac{n^2+n-2}{2} \\ &= \frac{(n+2)(n-1)}{2}.\end{aligned}$$

How do we even interprete this formula? Note that $n+2$ is no other than the first term plus the last term in the summand, while $(n-1)$ is the number of terms. Therefore, to add consecutive integers, we deduce that the sum should be

$$\frac{(\text{1st term} + \text{last term})(\text{number of terms})}{2}.$$

This concept works also for the original formula for $\sum_{k=1}^{n} k = \frac{(n+1)n}{2}$ (right?)

(2)

$$\sum_{k=1}^{n} (k+1)^2 = ?$$

Solution. Here, you should try converting to a case where the summand is k^2 since you know something about it. To go from $k+1$ to k, you are in fact shifting to the right, and thus, your index must increase by 1 also, that is,

$$\sum_{k=1}^{n} (k+1)^2 = \sum_{k=2}^{n+1} k^2.$$

Now, we need to check how many terms we are missing from the known formula $\sum_{k=1}^{n} k^2$. We note that in $\sum_{k=2}^{n+1}$, we do not count the case of $k=1$, but we added a case of $k=n+1$, relative to

$\sum_{k=1}^{n} k^2$. Therefore,

$$\sum_{k=2}^{n+1} k^2 = \left(\sum_{k=1}^{n} k^2\right) - 1^2 + (n+1)^2$$
$$= \frac{n\,(n+1)\,(2n+1)}{6} + (n+1)^2 - 1$$

Pretty ugly but you usually know what n is to compute the sum – or this should help you nail down the expression for some area under the curve using n subintervals.